Analysis and Design of a 3-DOF Flexure-based Zero-torsion Parallel Manipulator for Nano-alignment Applications

Guilin Yang, Tat Joo Teo, I-Ming Chen, and Wei Lin

Abstract—A flexure-based parallel manipulator (FPM) is a closed-loop compliant mechanism in which the moving platform is connected to the base through a number of flexural legs. Utilizing parallel-kinematics configurations and flexure joints, the FPMs can achieve extremely high motion resolution and accuracy. In this work, we focus on the analysis and design of a 3-DOF \((\theta_x - \theta_y - Z)\) zero-torsion FPM for nano-alignment applications. Among various possible zero-torsion parallel-kinematics configurations, it is identified that the 3-legged Prismatic-Prismatic-Spherical (3PPS) is a suitable candidate. Based on the concept of instantaneous rotation, the critical kinematic design issues, such as displacement and workspace analyses, are addressed. With these analysis algorithms, the major kinematic parameters are readily determined to meet the task requirements. To achieve a large workspace, beam-based flexure joints are employed in the FPM design. As the beam-based Universal (U) flexure joints are able to accommodate the required passive prismatic and spherical motions, each flexure PPS leg can be replaced by a simple flexure PU leg. A research prototype of the 3-DOF 3PU FPM has been developed, which achieves position and orientation resolutions of 20 \(\text{nm}\) and 0.05 \(\text{arcsecond}\) throughout a workspace of \(5^\circ \times 5^\circ \times 5\text{mm}\), respectively.

I. INTRODUCTION

A flexure-based parallel manipulator (FPM) is a closed-loop compliant mechanism in which the moving platform is connected to the base through a number of flexural legs. Benefitting from the advantages of both flexure joints and parallel-kinematics (closed-loop) configurations, the FPMs bear the essential features of nano-manipulators such as frictionless motion, absence of mechanical play and backlash, and insensitive to thermal variations and mechanical disturbances [1]. Despite their small workspace and limited payload, FPMs are still the most effective solution for various nano-positioning and nano-alignment applications [2], [3], [4], [5], [6], [7].

Ryu et al. [2] developed an \(X-Y-\theta_z\) planar-motion FPM that achieved a positioning resolution of \(8\text{nm}\) along the \(X\)- and \(Y\)-axes, and a rotational resolution of 0.057 \(\text{arcsecond}\) about the \(Z\)-axis. Lee and Kim [3] constructed another form of 3-DOF planar-motion FPM for wafer alignment and achieved a resolution of \(10\text{nm}\) and 0.2 \(\text{arcsecond}\) in the translational and rotational axes respectively. Other than the planar-motion FPMs, spatial-motion FPMs were also well demonstrated in past literatures. Tanikawa et al. [4] developed a spatial 3-DOF translational FPM based on 3RPPR configuration, which has a positioning accuracy of less than 100 nm for all three axes. Using the 6SPS parallel-kinematics configuration, Oiwa et al. presented a 6-DOF flexure-based Steward-Platform manipulator, which has a translational accuracy of 160 nm and rotational accuracy of 0.4 \(\text{arcsecond}\)[5].

A noticeable characteristic of these FPMs is that they have very limited workspace (in hundreds of micrometers and hundreds of arcseconds) and demand large actuating forces, resulting from the notched-hinge type flexure joints that have limited deflection range and high stiffness in driving directions. To achieve larger workspace, the latest trend in the FPM design is to employ the beam-based flexure joints with large deflection characteristics. A well-known example of a spatial-motion FPM is presented by Henein [6]. Based on the Delta robot configuration, the FPM achieves an \(X\)- \(Y\)- \(Z\) translational motion with a positioning repeatability of 100 \(\text{nm}\) and a 10 \(\text{mm}^3\) workspace. Similarly, Helmer [7] constructed a 6-DOF FPM using beam-based flexure joints, which achieves a workspace of \(\pm 5\text{ mm}\) in all translations and \(\pm 5^\circ\) in all rotations with positioning repeatability of 100 \(\text{nm}\) and 0.02 \(\text{arcsecond}\). Despite such research efforts, it is still a challenging issue to develop multi-DOF nano-positioning FPMs with large workspace (e.g., a few millimeters/degrees) and high payload (e.g., more than 100N).

In this work, a 3-DOF \((\theta_x - \theta_y - Z)\) zero-torsion FPM is to be developed for nano-imprinting (UV-embossing) tool modules for both coplanarity alignment and pressing force control. The stringent design requirements, such as a few degrees of coplanarity alignment about \(X\)- and \(Y\)-axes with sub-arcsecond resolution, a few millimeter translation along \(Z\) direction with nanometer resolution, and direct pressing force control of up to 100 \(N\), make the FPM design a very challenging task. Among the possible zero-torsion parallel-kinematics configurations, the 3-legged Prismatic-Prismatic-Spherical (3PPS) configuration is selected for the FPM design due to its great potential in achieving the designated performances. A geometrical modeling approach based on the concept of instantaneous rotation is employed for the displacement and workspace analyses. Such analysis algorithms are essential to evaluate the kinematic performance and determine the major design parameters. Based on the kinematic design results and utilizing the beam-based flexure joints, a research prototype the 3-DOF \((\theta_x - \theta_y - Z)\) FPM has been developed, which achieves position and orientation resolutions of 20 \(\text{nm}\) and 0.05 \(\text{arcsecond}\), respectively, and a continuous output force of 150N, throughout a workspace of \(5^\circ \times 5^\circ \times 5\text{mm}\). With such promising performances, this...
3-DOF FPM is currently employed for UV nanoimprint lithography applications [8].

II. PARALLEL-KINEMATICS CONFIGURATIONS

In order to develop a 3-DOF FPM to achieve the designated zero-torsion \( \theta_1 - \theta_2 - Z \) motions, the possible parallel-kinematics configurations need to be investigated. According to the literatures [9], [10], [11], the general considerations for designing a symmetric and compact 3-DOF zero-torsion \( \theta_1 - \theta_2 - Z \) parallel-kinematics configurations are listed as follows:

1) Three types of joints will be considered, i.e., 1-DOF Revolute (R) joint, 1-DOF Prismatic (P) joint, and 3-DOF Spherical (S) joint.
2) The parallel-kinematics configuration should have three identical legs to support its mobile platform.
3) Each leg should be a 5-DOF kinematic chain with a 1-DOF active prismatic or revolute joint.
4) To reduce the moving mass and moment of inertia, the active joint will be placed close to the base.
5) To reduce the number of passive joints, a passive 3-DOF spherical joint will be employed in each leg and it will be placed at the distal end of the leg.
6) To generate the 3-DOF \( \theta_1 - \theta_2 - Z \) motion, the possible motions of center of the spherical joint in each leg must be within a vertical plane. The three vertical planes, which are normally placed 120° apart for a symmetric design, must intersect at the same vertical line.

Based on these design considerations, we can enumerate the possible parallel-kinematics configurations, i.e., 3RRS, 3PRS, 3RPS, and 3PPS. Considering the precision motion requirements and the kinetostatic performance of the flexure joints, the 3PPS configuration is finally selected. In general, to design a high-precision mechanism, prismatic joints with stiff and precision guide ways are always preferred although they may result in a relative bulky structure. The same rule also applies to the FPM design as the flexure prismatic joints can produce precision motions with high off-axis stiffness and small parasitic errors.

For the 3PPS parallel-kinematics configuration, there are two possible locations for the active prismatic joint in each leg, i.e., to place it either at the first or the second joint locations (starting from the base). In this design, we place it at the first joint location in each leg, as shown in Fig. 1. Furthermore, to effectively make use of the limited strokes of both active and passive prismatic joints in each leg, the two prismatic joints in the same leg are placed perpendicular to each other. To be more specific, the active prismatic joint in each leg is placed vertically, while the passive prismatic joint is placed horizontally with its motion direction always pointing to the Z-axis of the base frame (which is located at center of the equilateral triangular base plate).

III. DISPLACEMENT ANALYSIS

The purpose of displacement analysis is to determine the displacement relationship between the active joints and the moving platform. Two displacement analysis issues, i.e., the forward and inverse displacement analysis, will be addressed in this section. In spite of the existing analysis approaches [12], [13], [14], [15], [16], a geometrical modeling method based on the concept of instantaneous rotation is proposed. As shown in Fig. 2, the equilateral triangular base plate is defined by the three leg’s attachment points, i.e., \( B_1, B_2, \) and \( B_3 \). The base coordinate frame, i.e., frame \( B \), is fixed at the center of the base plate with its Z-axis perpendicular to the base plate and X-axis parallel to \( B_2B_3 \).

To describe the motion of the moving platform, a local coordinate frame, i.e., frame \( P \), is attached to the center of the equilateral triangular moving plate, which is defined by the centers of the three spherical joints, i.e., \( P_1, P_2, \) and \( P_3 \). Similar to the definition of the base frame, the Z′-axis of the moving platform frame is parallel to the moving plate and its X′-axis is parallel to \( P_2P_1 \). Let \( a \) be the edge length of the equilateral triangular moving plate. Then the local coordinates of points \( P_i(i = 1, 2, 3) \) with respect to the moving platform frame \( P \) are: \( p'_1 = (0, \sqrt{3}/2a, 0) \), \( p'_2 = (-\frac{1}{2}a, -\sqrt{3}/6a, 0) \), and \( p'_3 = (\frac{1}{2}a, -\sqrt{3}/6a, 0) \). It is noted that the edge length of the base plate has no effect on the kinematics of the manipulator.
As the 3PPS parallel-kinematics configuration is a 3-DOF zero-torsion \( \theta_x-\theta_y-Z \) mechanism [20], the moving platform motions resulting from the three independent active prismatic joints displacements can be described by three independent kinematics parameters, although six parameters are generally needed to describe the moving platform pose. In the following paragraphs, it is realized that the displacement analysis is significantly simplified by appropriately selecting the three independent parameters of moving platform.

A. Forward Displacement Analysis

The purpose of forward displacement analysis is to determine the moving platform pose with three active prismatic joint displacements. As shown in Fig. 2, let the three active prismatic joint displacements be \( z_1, z_2, \) and \( z_3 \). Then the global coordinates of \( P_i (i = 1, 2, 3) \) with respect to base frame \( B \) can be written as: \( p_1 = (x_1, y_1, z_1), p_2 = (x_2, y_2, z_2), \) and \( p_3 = (x_3, y_3, z_3) \). Since the three passive prismatic joints have fixed moving directions which are always parallel to the base plate and placed 120\(^\circ\) apart, we can readily derive the following coordinates relationship: \( x_1 = 0, x_2 = \sqrt{3}y_2, \) and \( x_3 = -\sqrt{3}y_3. \)

As shown in Fig. 2, let the forward kinematic transformation from the base frame \( B \) to the moving platform frame \( P \) be \( T_{BP} \in SE(3) \), then

\[
T_{BP} = \begin{bmatrix} p_i' \end{bmatrix} = \begin{bmatrix} p_i \end{bmatrix}, \quad (i = 1, 2, 3). \quad (1)
\]

Since both frames \( B \) and \( P \) are right-hand Cartesian coordinate frames, we also have

\[
T_{BP} \left[ \begin{array}{c} p_{12}' \\ p_{23}' \end{array} \right] = \left[ \begin{array}{c} p_{12} \\ p_{23} \end{array} \right], \quad (2)
\]

where \( p_{12}' = p_2 - p_1, p_{23}' = p_3 - p_2, p_{12} = p_2 - p_1, \) and \( p_{23} = p_3 - p_2. \) Combining Eqs. (1) and (2), the pose of the end-effector frame, \( T_{BP} \), can be given by

\[
T_{BP} = \begin{bmatrix} p_1 & p_2 & p_3 & p_{12} & p_{12}' & p_{23}' & p_{12} & p_{23} \end{bmatrix} \left[ \begin{array}{c} \begin{bmatrix} p_1' \\ p_2' \\ p_3' \end{bmatrix} \\ \begin{bmatrix} p_{12}' \\ p_{23}' \end{bmatrix} \end{array} \right]^{-1}. \quad (3)
\]

Note that the matrix whose inverse appears in Eq. (3) is a constant matrix for a specific design, and is always invertible if points \( P_1, P_2, \) and \( P_3 \) neither coincide nor collinear such that \( p_{12} \times p_{23} \neq 0. \)

As \( T_{BP} \in SE(3) \), it always has the form of: \( T_{BP} = \begin{bmatrix} R_{BP} & P_{BP} \\ 0 & 1 \end{bmatrix} \), where \( R_{BP} \in SO(3) \) and \( P_{BP} \in \mathbb{R}^{3 \times 1} \) represent the orientation and position of frame \( P \) with respect to frame \( B \), respectively. After a symbolic computation of \( T_{BP} \), we have:

\[
R_{BP} = \begin{bmatrix} \begin{array}{c} -\frac{\sqrt{3}(y_1+y_2)}{\sqrt{2}y_3} \\ -\frac{\sqrt{3}y_1}{\sqrt{2}y_3} \\ \frac{\sqrt{3}y_2}{\sqrt{2}y_3} \end{array} \\ \begin{array}{c} \frac{\sqrt{3}(y_1+y_2)}{\sqrt{2}y_3} \\ \frac{\sqrt{3}y_1}{\sqrt{2}y_3} \\ \frac{\sqrt{3}y_2}{\sqrt{2}y_3} \end{array} \\ \begin{array}{c} \frac{\sqrt{3}y_1}{\sqrt{2}y_3} \\ \frac{\sqrt{3}y_2}{\sqrt{2}y_3} \\ \frac{\sqrt{3}y_1}{\sqrt{2}y_3} \end{array} \end{bmatrix} \quad (4)
\]

\[
P_{BP} = \begin{bmatrix} \frac{\sqrt{3}y_3}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}(y_1+y_2+y_3) \\ \frac{1}{\sqrt{2}}(z_1+z_2+z_3) \end{bmatrix}^T \quad (5)
\]

By examining the symbolic expression of \( R_{BP} \) in Eq. (4), we realized that the entry in the 1\(^{st} \) row and 2\(^{nd} \) column is equal to the entry in the 2\(^{nd} \) row and 1\(^{st} \) column, i.e., \( R_{BP}(1, 2) = R_{BP}(2, 1) \), which implies the unique orientation characteristics of this parallel-kinematics configuration, i.e., zero-torsion motion [15]. To interpret such an interesting result, let us recall that any 3D orientation (represented by a 3 \( \times \) 3 rotation matrix) can be realized by a rotation about a unit vector \( \omega = (\omega_x, \omega_y, \omega_z) \) \( (\| \omega \| = 1) \) of an angle \( \theta \), which is given by [17, 18]:

\[
R = e^{\theta \omega} = \begin{bmatrix} 1 - (\omega_y^2 + \omega_z^2) & \omega_x \omega_y(1 - \omega_z) - \omega_z \omega_x & \omega_x \omega_z(1 - \omega_y) + \omega_y \omega_x \\ \omega_x \omega_y(1 - \omega_z) + \omega_z \omega_x & 1 - (\omega_x^2 + \omega_z^2) & \omega_y \omega_x(1 - \omega_z) - \omega_z \omega_y \\ \omega_x \omega_z(1 - \omega_y) - \omega_y \omega_x & \omega_y \omega_x(1 - \omega_z) + \omega_z \omega_y & 1 - (\omega_x^2 + \omega_y^2) \end{bmatrix} \quad (6)
\]

where \( \omega = (1 - \cos \theta) \). In the above rotation matrix, if \( R(1, 2) = R(2, 1) \) when \( \theta \neq 0 \), we can readily determine that \( \omega_x = 0 \), which implies that the rotation is about an axis which is parallel to the \( XY \) plane of the reference frame. Let \( \omega_x = 0 \), Eq. (6) can be simplified as:

\[
R = \begin{bmatrix} 1 - \omega_y^2 & \omega_x \omega_y & \omega_x \sin \theta \\ \omega_x \omega_y & 1 - \omega_z^2 & -\omega_x \sin \theta \\ -\omega_x \sin \theta & \omega_x \sin \theta & 1 - (\omega_x^2 + \omega_y^2) \end{bmatrix} \quad (7)
\]

From Eq. (7), we can further obtain: \( R(3, 1) = -R(1, 3) \) and \( R(3, 2) = -R(2, 3) \). Such interesting relationships will significantly simplify the displacement analysis issue.

Fig. 3. Rotation about an axis parallel to \( XY \) plane
less than $\frac{\pi}{2}$, $\varepsilon_z$ can be uniquely determined by $e_x$ and $e_y$ such that: $e_z = \sqrt{1 - e_x^2 - e_y^2}$. Hence, the rotation vector $\omega$ can be written as:

$$\omega = \frac{Z \times Z'}{\|Z \times Z'\|} = (-\frac{e_y}{\sqrt{e_x^2 + e_y^2}}, \frac{e_x}{\sqrt{e_x^2 + e_y^2}}, 0)$$ (8)

Furthermore, instead of directly determining the rotation angle $\theta$, we can readily obtain the expressions of $\sin \theta$ and $\cos \theta$ respectively as:

$$\sin \theta = \|Z \times Z'\| = \sqrt{e_x^2 + e_y^2}$$ (9)
$$\cos \theta = Z \cdot Z' = 1 - e_x^2 - e_y^2$$ (10)

Substituting Eqs. (8)-(10) into Eq. (7), the moving platform orientation is given by:

$$R_{BP} = \begin{bmatrix}
\frac{e_x^2 + e_y^2}{e_x^2 + e_y^2} & \frac{e_x e_y (1 + e_z)}{e_x^2 + e_y^2} & e_x \\
-e_y & \frac{e_x e_y (1 - e_z)}{e_x^2 + e_y^2} & e_y \\
0 & 0 & e_z
\end{bmatrix}$$ (11)

where $e_z = \sqrt{1 - e_x^2 - e_y^2}$. By comparing Eq. (11) with Eq. (4), we obtain:

$$e_x = \frac{z_2 - z_3}{a}$$ (12)
$$e_y = -\frac{\sqrt{3}(z_2 - z_1 - z_3)}{3a}$$ (13)

From Eq. (5), it is apparent that

$$z_p = \frac{z_1 + z_2 + z_3}{3}$$ (14)

Equations (12), (13), and (14) are the forward displacement solutions for the 3PPS configuration. They are the linear functions of the three active prismatic joint displacements, i.e., $z_1$, $z_2$, and $z_3$, and can be uniquely determined if the rotation angle $\theta$ is less than $\frac{\pi}{2}$.

**B. Inverse Displacement Analysis**

The purpose of inverse displacement analysis is to determine three active prismatic joint displacements with the known moving platform pose. As shown in Fig. 2, when the three independent kinematics parameters of the moving platform, i.e., $e_x$, $e_y$, and $z_p$, are known, the three active prismatic joint displacements can be readily determined from Eq. (12), (13), and (14) as follows:

$$z_1 = -\frac{\sqrt{3}a e_x}{3} + z_p$$ (15)
$$z_2 = \frac{3a e_x + \sqrt{3}a e_y}{6} + z_p$$ (16)
$$z_3 = \frac{-3a e_x + \sqrt{3}a e_y}{6} + z_p$$ (17)

Equations (15), (16), and (17) are the inverse displacement solutions for the 3PPS configuration. They are also the linear functions of the three independent kinematics parameters of the moving platform, i.e., $e_x$, $e_y$, and $z_p$. They have unique solutions if the rotation angle $\theta$ is less than $\frac{\pi}{2}$.

**C. Determination of the Kinematic Design Parameters**

In this 3-DOF 3PPS zero-torsion FPM design, two major kinematic design requirements, i.e., the tilting angle about the horizontal plane and the vertical translational displacement, need to be satisfied. Based on the application needs of the nano-imprinting (UV-embossing) tool modules, the tilting angle and vertical translational displacement of the moving platform are $\pm 2^\circ$ and $4\text{mm}$, respectively. According to the notations adopted in this paper, the tilting angle (as shown in Fig. 3) is represented by the rotation angle $\theta$ about $\omega$, i.e., $\theta \in [0, \pi]$, while the vertical translational displacement is represented by $z_p$, i.e., $z_p \in [0, 4\text{mm}]$.

Based on these two task requirements, the major kinematic design parameters to be determined are the edge length of the equilateral triangular moving plate and the stroke of the active prismatic joints. With Eqs. (12), (13), and (14), it is apparent that a larger active prismatic joint stroke and a smaller edge length of the moving plate will achieve a larger tilting angle and a larger vertical displacement. Furthermore, to obtain $4\text{mm}$ vertical displacement, the stroke of the active prismatic joint, based on Eq. (14), must be larger than $4\text{mm}$. Therefore, the long-stroke flexure-based electromagnetic linear actuator (FELA) is employed [19]. Now, let the stroke of the three active prismatic joints be $5\text{mm}$. Then the edge length $a$, based on Eqs. (12) and (13), must be smaller than $165\text{mm}$. Although a smaller $a$ is preferred for a compact design, the smallest value of $a$ will be constrained by the actual dimensions of the FELA. To achieve targeted thrust force of more than $100\text{N}$, the smallest value of $a$ must be more than $120\text{mm}$ based on the FELA design. Considering these two dimension limits of $a$, we eventually take $125\text{mm}$ for the edge length $a$.

According to the key design parameters determined, i.e., $125\text{mm}$ for the edge length of the triangular base moving plate and $5\text{mm}$ for the stroke of the active prismatic joints, the maximum tilting angle about the X- and Y-axis of base frame and vertical translational displacement of the moving platform, based on Eqs. (12), (13), and (14), are $\pm 2.65^\circ$ and $5\text{mm}$, respectively.

**IV. WORKSPACE ANALYSIS AND VISUALIZATION**

To validate the kinematic design parameters, a workspace analysis and visualization scheme has been proposed in this Section based on the displacement analysis algorithms. Three independent parameters are employed, i.e., $\theta \in [0, \pi]$, $\phi \in [0, 2\pi]$, and $z_p \in [z_{p_{\text{min}}}, z_{p_{\text{max}}}]$, to describe both the Z-axis direction (Fig. 3) and the vertical displacements of the moving platform frame. Here, $\theta$ and $\phi$ can be uniquely determined by $e_x$ and $e_y$, if $\theta < \frac{\pi}{2}$. To intuitively represent the 3-DOF workspace of the moving platform, cylindrical coordinates, i.e., $(\theta, \phi, z_p)$, are employed as shown in Fig. 4(a). Through a finite partition of the parametric workspace defined by the three independent cylindrical coordinates and utilizing the inverse displacement analysis algorithm for workspace elements detection, the resultant workspace based on the proposed kinematic design parameters is shown in Fig. 4(b). From this figure, it is shown that the maximum values of...
θ and \(z_p\) are about ±2.65° (i.e., 0.046 rad) and 5mm, respectively. Note that in this workspace analysis, the initial heights of the three active prismatic joints are all equal to 147.5mm and they vary from 147.5mm to 152.5mm with maximum displacement of 5mm.

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V. Prototype and Experimental Results

A. Prototype Development

Based on the kinematic analysis and design results presented in the previous Sections, the first version of the 3-DOF 3PPS FPM was designed, as shown in Fig. 5. It consists of a base plate, three flexure-based electromagnetic linear actuator (FELA) modules, three passive flexure prismatic joint modules, three passive flexure spherical joint modules, and a moving plate. In particular, each passive flexure spherical joint module is a combination of a flexure universal joint and a flexure rotatory joint to allow 3-DOF spherical motions. To have large deflections, all the joint modules were constructed with beam-based flexures. With the ANSYS software, Finite Element (FE) analysis was conducted on the 3-DOF 3PPS FPM design. The analysis results indicate that the required deflections for both the passive flexure prismatic joint modules and the rotatory joint portion of the passive flexure spherical joint modules are all very small and negligible. Based on such analysis results, we have further modified the FKM design by removing these two flexure joints from each leg. The 3PPS FPM thus becomes a 3PU FPM, thanks for the beam-based flexure universal joint modules that can also accommodate small prismatic and rotary deflections. After such design modification, a research prototype of the 3-DOF 3PU FPM has been developed, as shown in Fig. 6. It consists of a base plate, three FELA modules with built-in 5mm resolution MicroE encoders, three passive flexure universal joint modules, and a moving plate.

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B. Experimental Results

To evaluate the the performance of the 3-DOF 3PU FPM prototype, various experimental investigations have been conducted, which include workspace, motion resolution, and loading capacity evaluations.

The workspace of this FPM is evaluated by the Leica laser tracker system which consists of a tracker robot and a 6-DOF measurement device, T-MAC. In this set-up, T-MAC is mounted onto the moving platform of the FPM so that the laser tracker robot can trace and measure the 6-DOF motions of the moving platform. When the three FELAs are controlled to move within their 5mm stroke, the workspace measurement results are listed as follows:

- Vertical displacement: \(-2.500\text{mm}\) to \(2.514\text{mm}\)
- Tilting angle about X-axis: \(-2.868°\) to \(2.981°\)
- Tilting angle about Y-axis: \(-2.556°\) to \(2.578°\)

The above results show that this FPM prototype is able to achieve the targeted workspace.

As shown in Fig. 7(a), a Renishaw laser interferometer encoder with 10nm accuracy is employed to evaluated the translational motion resolution in vertical direction. The laser encoder is mounted on a fixed frame, while a metrology mirror is mounted on the top surface of the moving plate. Figure 7(b) the motion resolution measurement results from Renishaw laser interferometer, where the three FELAs are simultaneously controlled to achieve 20nm step motions vertically. From this figure, it can be seen that the motion resolution of the system is better than 20nm.
less than $\frac{\pi}{2}$, $e_z$ can be uniquely determined by $e_x$ and $e_y$ such that: $e_z = \sqrt{1 - e_x^2 - e_y^2}$. Hence, the rotation vector $\omega$ can be written as:

$$\omega = \frac{Z \times Z'}{\|Z \times Z'\|} = (-\frac{e_y}{\sqrt{e_x^2 + e_y^2}}, \frac{e_x}{\sqrt{e_x^2 + e_y^2}}, 0)$$ (8)

Furthermore, instead of directly determining the rotation angle $\theta$, we can readily obtain the expressions of $\sin \theta$ and $\cos \theta$ respectively as:

$$\sin \theta = \|Z \times Z'\| = \sqrt{e_x^2 + e_y^2}$$ (9)
$$\cos \theta = Z.Z' = \sqrt{1 - e_x^2 - e_y^2}$$ (10)

Substituting Eqs. (8)-(10) into Eq. (7), the moving platform orientation is given by:

$$R_{BP} = \begin{bmatrix}
\frac{e_y}{\sqrt{e_x^2 + e_y^2}} & \frac{e_x}{\sqrt{e_x^2 + e_y^2}} & 0 \\
\frac{e_x(1 + e_z)}{e_x^2 + e_y^2} & \frac{e_y(1 + e_z)}{e_x^2 + e_y^2} & e_z \\
-e_x & -e_y & e_z
\end{bmatrix}$$ (11)

where $e_z = \sqrt{1 - e_x^2 - e_y^2}$. By comparing Eq. (11) with Eq. (4), we obtain:

$$e_x = \frac{z_2 - z_3}{a}$$ (12)
$$e_y = -\frac{\sqrt{3}(2z_1 - z_2 - z_3)}{3a}$$ (13)

From Eq. (5), it is apparent that

$$z_p = \frac{z_1 + z_2 + z_3}{3}$$ (14)

Equations (12), (13), and (14) are the forward displacement solutions for the 3PPS configuration. They are the linear functions of the three active prismatic joint displacements, i.e., $z_1, z_2,$ and $z_3$, and can be uniquely determined if the rotation angle $\theta$ is less than $\frac{\pi}{2}$.

B. Inverse Displacement Analysis

The purpose of inverse displacement analysis is to determine the three active prismatic joint displacements with the known moving platform pose. As shown in Fig. 2, when the three independent kinematics parameters of the moving platform, i.e., $e_x, e_y,$ and $z_p$, are known, the three active prismatic joint displacements can be readily determined from Eq. (12), (13), and (14) as follows:

$$z_1 = \frac{-\sqrt{3}e_x}{3} + z_p$$ (15)
$$z_2 = \frac{3ae_x + \sqrt{3}ae_y}{6} + z_p$$ (16)
$$z_3 = \frac{-3ae_x + \sqrt{3}ae_y}{6} + z_p$$ (17)

Equations (15), (16), and (17) are the inverse displacement solutions for the 3PPS configuration. They are also the linear functions of the three independent kinematics parameters of the moving platform, i.e., $e_x, e_y,$ and $z_p$. They have unique solutions if the rotation angle $\theta$ is less than $\frac{\pi}{2}$.

C. Determination of the Kinematic Design Parameters

In this 3-DOF 3PPS zero-torsion FPM design, two major kinematic design requirements, i.e., the tilting angle about the horizontal plane and the vertical translational displacement, need to be satisfied. Based on the application needs of the nano-imprinting (UV-embossing) tool modules, the tilting angle and vertical translational displacement of the moving platform are $\pm 2^\circ$ and $4\text{mm}$, respectively. According to the notations adopted in this paper, the tilting angle (as shown in Fig. 3) is represented by the rotation angle $\theta$ about $\omega$, i.e., $\theta \in [0, 2^\circ]$, while the vertical translational displacement is represented by $z_p$, i.e., $z_p \in [0, 4\text{mm}]$.

Based on these two task requirements, the major kinematic design parameters to be determined are the edge length of the equilateral triangular moving plate and the stroke of the active prismatic joints. With Eqs. (12), (13), and (14), it is apparent that a larger active prismatic joint stroke and a smaller edge length of the moving plate will achieve a larger tilting angle and a larger vertical displacement. Furthermore, to obtain $4\text{mm}$ vertical displacement, the stroke of the active prismatic joint, based on Eq. (14), must be larger than $4\text{mm}$. Therefore, the long-stroke flexure-based electromagnetic linear actuator (FELA) is employed [19].

Now, let the stroke of the three active prismatic joints be $5\text{mm}$. Then the edge length $a$, based on Eqs. (12) and (13), must be smaller than $165\text{mm}$. Although a smaller $a$ is preferred for a compact design, the smallest value of $a$ will be constrained by the actual dimensions of the FELA. To achieve targeted thrust force of more than $100\text{N}$, the smallest value of $a$ must be more than $120\text{mm}$ based on the FELA design. Considering these two dimension limits of $a$, we eventually take $125\text{mm}$ for the edge length $a$.

According to the key design parameters determined, i.e., $125\text{mm}$ for the edge length of the triangular base moving plate and $5\text{mm}$ for the stroke of the active prismatic joints, the maximum tilting angle about the X- and Y-axis of base frame and vertical translational displacement of the moving platform, based on Eqs. (12), (13), and (14), are $\pm 2.65^\circ$ and $5\text{mm}$, respectively.

IV. WORKSPACE ANALYSIS AND VISUALIZATION

To validate the kinematic design parameters, a workspace analysis and visualization scheme has been proposed in this Section based on the displacement analysis algorithms. Three independent parameters are employed, i.e., $\theta \in [0, \pi]$, $\phi \in [0, 2\pi]$, and $z_p \in [z_{pmin}, z_{pmax}]$, to describe both the Z-axis direction (Fig. 3) and the vertical displacements of the moving platform frame. Here, $\theta$ and $\phi$ can be uniquely determined by $e_x$ and $e_y$, if $\theta < \frac{\pi}{2}$. To intuitively represent the 3-DOF workspace of the moving platform, cylindrical coordinates, i.e., $(\theta, \phi, z_p)$, are employed as shown in Fig. 4(a). Through a finite partition of the parametric workspace defined by the three independent cylindrical coordinates and utilizing the inverse displacement analysis algorithm for workspace elements detection, the resultant workspace based on the proposed kinematic design parameters is shown in Fig. 4(b). From this figure, it is shown that the maximum values of